

Decoherence and the Quantum Zeno Effect

Anu Venugopalan* and R. Ghosh

*School of Physical Sciences,
Jawaharlal Nehru University, New Delhi - 110 067, INDIA.*

The measurements in the optical test of quantum Zeno effect [Itano et al., Phys. Rev. A **41** (1990) 2295] are analyzed using the environment-induced decoherence theory, where the spontaneous emission lifetime of the relevant level emerges as the ‘decoherence time’. The implication of this finite decoherence time in setting a fundamental limit on the realizability of the condition of continuous measurements is investigated in detail.

According to the original formulation by Misra and Sudarshan [1], the quantum Zeno effect (QZE) refers to the inhibition of the temporal evolution of a dynamical system under continuous observation during a period of time. Misra and Sudarshan considered the example of an unstable particle and showed that under ‘continuous’ measurements, the particle will never be found to decay [1]. Their analysis does not take into account the actual mechanism of the measurement process involved, but is based on an alternating sequence of unitary time evolution and a ‘collapse’ of the wave function based on the projection postulate of von Neumann [2]. In performing *continuous* measurements, it is assumed that successive measurements are instantaneous, yet independent [1]. Ghirardi et al. [3] have shown that when the time-energy uncertainty relations are taken into account, the dependence of the measured lifetime on the frequency of measurements, although present in principle, would be extremely difficult to observe in the case of spontaneous decay. The QZE has not yet been observed experimentally for spontaneous decay.

Following an original proposal by Cook [4], Itano et al. [5] demonstrated experimentally the occurrence of the QZE as the inhibition of *induced* radio frequency transitions between two hyperfine levels of a berillium ion caused by frequent measurements of the level population using optical pulses (see Fig. 1). This test of QZE does not involve observation of spontaneous decay of naturally unstable quantum system, as in the original idea by Misra and Sudarshan. The system is initially prepared in level 1. The $1 \rightarrow 2$ transition is driven by a resonant radio-frequency π pulse. In order to observe the level populations, level 1 is connected by an optical transition to an additional level 3 such that level 3 can decay only to level 1. Spontaneous decay from level 2 to level 1 is negligible. The measurements are carried out (during the evolution under the π pulse) by driving the $1 \rightarrow 3$ transition with N equispaced short optical pulses and observing the presence (or absence) of spontaneously emitted photons from level 3 corresponding to the atom being found in the level 1 (or 2). In the experiment in ref. [5], a ‘freezing’ of population in one level was observed

as a result of frequent measurements. There have been many attempts to theoretically explain this experimental observation of the QZE [5-7], and to classify the various examples of quantum Zeno effect (or ‘paradox’) in terms of either strictly unitary evolutions or ‘wave-function collapse’ by measurement.

Figure 1 *Energy-level diagram for the quantum Zeno effect experiment [5]. In our model, the environment enters as the collection of vacuum modes coupled to level 3.*

In this Letter we apply the environment-induced decoherence theory [8] to analyze the observed results in the QZE experiment [5], and provide connection with the theoretical analysis in ref. [6]. We demonstrate that a fundamental limit on the frequency of measurements emerges from the theory, which is in tune with the predictions in ref. [3], and can be contrasted with the recent findings in ref. [7] in the context of QZE with neutron spin.

By using the postulate of projection of the wave function [2] Itano et al. have shown that in the limit of infinitely frequent measurements, the probability of one of the levels being populated goes to unity in the following way [5]. Suppose the ion is in level 1 at time $\tau = 0$. An on-resonance π -pulse of duration $T = \pi/\Omega$ is applied, where Ω is the Rabi frequency. Without the measurement pulses, the probability $P_2(T)$ for the ion to be in level 2 at $\tau = T$ is 1. Let N measurement pulses be applied within T at times $\tau_k = kT/N$, $k = 1, 2, \dots, N$. The level populations at the end of the π pulse are calculated using the Bloch vector representation of a two-level system [5]. In the rotating wave approximation, the system can be described by a Bloch vector $\mathbf{R} \equiv (R_1, R_2, R_3)$, whose components in terms of the density matrix elements are

$$\begin{aligned} R_1 &\equiv \rho_{12} + \rho_{21}, \\ R_2 &\equiv i(\rho_{12} - \rho_{21}), \\ R_3 &\equiv \rho_{22} - \rho_{11} \equiv P_2 - P_1, \end{aligned} \quad (1)$$

where $P_1(t)$ and $P_2(t)$ are the probabilities for the ion to be in the levels 1 and 2, respectively. The equation of motion for \mathbf{R} is

$$\frac{d\mathbf{R}}{dt} = \omega \mathbf{X} \mathbf{R}, \quad (2)$$

where $\omega = (\Omega, 0, 0)$. The *projection postulate* for each measurement essentially amounts to setting the coherences (ρ_{12}, ρ_{21}) to zero while leaving the populations

(ρ_{11}, ρ_{22}) unchanged. It can be easily seen [5] that after N measurements (projections) the population $P_2(T) = [R_3(T) + 1]/2$ at the end of the π pulse is

$$P_2(T) = [1 - \cos^N(\pi/N)]/2. \quad (3)$$

For large N ,

$$\begin{aligned} P_2(T) &\cong \lim_{N \rightarrow \infty} [1 - (1 - \pi^2/2N^2)^N]/2 \\ &\cong \lim_{N \rightarrow \infty} [1 - \exp(-\pi^2/2N)]/2 \cong 0. \end{aligned} \quad (4)$$

As $N \rightarrow \infty$, $P_2(T)$ decreases monotonically to zero. This, according to Itano et al. [5], is the explanation of the observed Zeno effect.

In another approach, Frerichs and Schenzle [6] have shown that the outcome of the experiment in ref. [5] can be explained by looking at the optical Bloch equations for the complete *three*-level system. These semiclassical equations of motion for the elements of the density matrix take into account the dissipation effects due to spontaneous emission along the optical transition path. Under the condition that the spacing T/N of the measurement pulses is greater than the spontaneous emission lifetime of the third level, it has been demonstrated by numerical integration of the Bloch equations [6] that all measurable results of quantum Zeno experiment emerge naturally from continuous irreversible quantum dynamics, without having to appeal to concepts such as the projection postulate or the wave function collapse. A number of recent studies attempt to explain the observed inhibition of transitions using the unitary evolution given by the Schrodinger equation, but provide no specific mechanism for the measurement process.

We propose that the measurement processes involved in the experiment of Itano et al. [5] can be explained by the environment-induced decoherence approach [8]. This approach is based on the understanding that a macroscopic apparatus is never isolated from its environment. Note that the actual measurement process in the QZE experiment [5] involves the observation (or non-observation) of *spontaneously* emitted photons from level 3 (see Fig.1). While the irreversible processes of spontaneous emission are accounted for by phenomenological decay rates in the semiclassical theory, a fully quantized theory of the field and atom is required to explain it. It is well known in the Weisskopf-Wigner theory [9] that spontaneous emission decay rates emerge naturally when a completely quantized field treatment including the coupling to the field vacuum modes, i.e., *the environment*, is considered. Since one is interested only in the dynamics of the atom, an average over the large number of degrees of freedom of the multimode vacuum leads to an irreversible dynamic process. The reduced density matrix of the atom is, thus, driven to a diagonal form over a time scale equal to the spontaneous emission lifetime of the excited level. This treatment is exactly that of the ‘environment- induced decoherence’ approach [8]. The experiment of Itano et al. [5] can now be viewed in the

following way. The two-level system (levels 1 and 2) constitutes the ‘system’ of interest for which QZE is to be investigated. Level 3 is coupled to the system of interest in such a way that it can be used to measure the level populations (i.e., to determine whether the ion is in level 1 or 2) and hence level 3 can be interpreted as the ‘apparatus’. The collection of vacuum modes is the ‘environment’ the effect of which is significant only for level 3 (see Fig. 1) as it has a finite spontaneous emission lifetime. Since level 2 is assumed to be metastable, the system is coupled only to the apparatus and is isolated from the environment. The system-apparatus interaction is through the short optical pulses that connect levels 1 and 3. The equation for the reduced ‘system-apparatus’ composite after tracing over the environment variables are the optical Bloch equations considered by Frerichs and Schenzle [6]. The interaction with the environment drives the density matrix of the apparatus to a diagonal form. Since the apparatus is in turn coupled to the system, the density matrix of the system-apparatus composite acquires a diagonal form. The environment- induced decoherence approach provides a mechanism for *deriving* the phenomenological damping in the Bloch equations in the way described above.

The physical meaning and realizability of the ‘continuous measurements’ limit as incorporated in the example of spontaneous decay of an unstable particle in the original formulation of the Zeno problem was criticised by Ghirardi et al. [3] using arguments based on the time-energy uncertainty relations. In the following we show that in the environment-induced decoherence theory, there exists a fundamental limit on the maximum number of measurements that can be performed in a set-up similar to that in ref. [5]. The crucial point here is that the *decoherence* of the reduced density-matrix of the system-apparatus (when the density matrix collapses to a diagonal form) does not take place instantaneously, but over a characteristic time scale - the *decoherence time*. It is obvious that the decoherence time for each measurement is the spontaneous emission lifetime of the level 3. This sets a fundamental limit on the requirement of ‘continuous measurements’ for the QZE since the photons from level 3 cannot be observed at a rate faster than the decoherence rate. The limit of infinitely frequent or continuous measurements is then a mathematical idealization. Let us consider the analysis of Itano et al. again. The measurements are made at times $\tau_k = kT/N$, $k = 1, 2, \dots, N$, where for a π -pulse, $T = \pi/\Omega$, Ω being the Rabi frequency. The probability, $P_2(T)$, of the ion being in level 2 as given by (3) monotonically decreases to zero as $N \rightarrow \infty$. However, since the time between the measurements is limited by the spontaneous emission lifetime τ_{sp} of the third level, there exists a *lower bound* on the argument of the cosine term in (3), viz.,

$$\pi/N \geq \Omega\tau_{sp}. \quad (5)$$

Therefore, for large N , the expression (4) for $P_2(T)$ be-

comes

$$P_2(T) \cong \lim_{N \rightarrow \infty} [1 - (1 - \Omega^2 \tau_{sp}^2 / 2)^N] / 2 \\ \cong \lim_{N \rightarrow \infty} [1 - \exp(-\Omega^2 \tau_{sp}^2 N / 2)] / 2. \quad (6)$$

In the limit of $N \rightarrow \infty$, the above expression *does* not tend to zero, but approaches one-half. As explained earlier, the theoretical analysis of Itano et al. assumes the artificial collapse hypothesis with no dynamical mechanism for the measurement, and Eq.(3) is said to be the explanation of the observed effect in the limit of $N \rightarrow \infty$. If one tries to incorporate the finite measurement time into the analysis of Itano et al., it leads to the paradoxical situation described by Eq.(6). From (5) it is obvious that N cannot increase indefinitely to infinity but is limited to N_{max} , where

$$N_{max} \equiv \pi / (\Omega \tau_{sp}) = T / \tau_{sp}. \quad (7)$$

The origin of this limitation on the maximum value of N is the finite spontaneous emission lifetime of the level 3, and hence can be traced back to the time-energy uncertainty relation [3].

In a real experiment, increasing the number of measurement pulses N beyond N_{max} would amount to making the $1 \rightarrow 3$ transition almost continuous. Since a measurement is defined solely by the observation or non-observation of spontaneously emitted photons from level 3, the process of measurement of level population would be ill-defined in such a situation since level 3 would not be allowed to decay to level 1. For $N > N_{max}$, the population gets stuck in level 3, and the non-observation of photons from level 3 here does not imply that the population is stuck in level 2 as is the case in QZE. Thus increasing N beyond N_{max} is unsuited from the point of view of the QZE.

Recently, Nakazato et al. [7] have studied the QZE in the case of neutron spins which is similar in spirit to the system studied by Itano et al. [5]. Nakazato et al. have shown that the limit of continuous measurements is unphysical. In their example, if the neutron is initially prepared in the spin-up (\uparrow) state in the presence of a magnetic field, it evolves to the spin-down state after a time T in a way similar to the two-level ion going from level 1 to level 2 on the application of a π pulse in Ref. [5]. Now, if one does N measurements of the state at regular time-intervals of T/N during T , the probability that the neutron spin is up at time T in the limit $N \rightarrow \infty$ is

$$P_{\uparrow}(T) = \lim_{N \rightarrow \infty} [\cos^2(\pi/2N)]^N \cong 1. \quad (8)$$

Here $\pi/2N = \mu B l / \hbar v$, where B is the applied static magnetic field, μ is the modulus of the neutron magnetic moment, l is the length of the region where B is present, v is the neutron speed and \hbar is Planck's constant. Nakazato et al. [7] argue that from a physical

point of view it is impossible to avoid uncertainties in the neutron speed Δv and the position Δx . They show that the argument $\phi \equiv \pi/2N$ of the cosine term has a lower bound, $\phi_0 = \Delta E_m / 4\Delta E_k$, where $\Delta E_m = 2\mu B$ is the magnetic energy gap, and $\Delta E_k = \Delta(mv^2/2)_{v=v_0}$ is the spread in the kinetic energy of the neutron beam at the mean speed v_0 . Thus, for large N ,

$$P_{\uparrow}(T) \cong (\cos \phi_0)^{2N} \cong (1 - \phi_0^2/2)^{2N} \cong \exp(-\phi_0^2 N). \quad (9)$$

Thus, as $N \rightarrow \infty$, $P_{\uparrow}(T)$ *vanishes to zero* unlike in (8). Nakazato et al. thus conclude that the limit $N \rightarrow \infty$ is unphysical as it leads to the paradoxical result (9) when the uncertainty relations are taken into account. Nakazato et al. [7] propose a limiting value of N which is obtained by setting $P_{\uparrow}(T) \cong 1/2$ in (9). Note that if we deduce the maximum value of N from the lower bound ϕ_0 of $\pi/2N$, so that

$$N_{max} = \pi / (2\phi_0) = 2\pi \Delta E_k / \Delta E_m, \quad (10)$$

then by a proper choice of N_{max} , $P_{\uparrow}(T)$ can be made to exceed $1/2$, and it can be very close to unity. Thus the assertion of $P_{\uparrow}(T) \cong 1/2$ at $N \cong N_{max}$ in [7] is not quite justified.

In summary, we have shown that the *irreversibility* in the quantum dynamical equations associated with the *measurements* in a quantum Zeno experiment [5] can be derived using the environment-induced decoherence approach in which the interaction of an atomic level with the vacuum modes of the quantized field results in spontaneous emission from the atomic level. In the experiment, the observation of spontaneously emitted photons from a level defines a measurement. The finite time taken by the atomic density matrix to *decohere* (and hence yield a measurement) under such an interaction with the vacuum modes is the spontaneous emission life-time of the atomic state concerned. This finite decoherence time limits the frequency of measurements in the quantum Zeno set-up. Our results are compared with those in the recently studied example of quantum Zeno effect in neutron spins [7].

RG acknowledges partial support from a grant by the Department of Science and Technology, Government of India.

* Present address:

Theoretical Physics Group, Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay - 400 005, India.
Electronic address: anu@theory.tifr.res.in

- [1] B. Misra and E.C.G. Sudarshan, J. Math. Phys. **18** (1977) 756.
- [2] J. von Neumann, in *Mathematische Grundlagen der Quantenmechanik* (Springer-Verlag, Berlin, 1932); English

- translation by R. T. Beyer (Princeton University Press, Princeton, 1955); Partly reprinted in *Quantum Theory and Measurements*, eds. J. A. Wheeler and W. H. Zurek (Princeton University Press, Princeton, 1983).
- [3] G. C. Ghirardi, C. Omero, T. Weber and A. Rimini, *Nuovo Cimento* **52A** (1979) 421.
 - [4] R. J. Cook, *Phys. Scr.* **T21** (1988) 49.
 - [5] W. M. Itano, D. J. Heinzen, J. J. Bollinger and D. J. Wineland, *Phys. Rev. A* **41** (1990) 2295.
 - [6] V. Frerichs and A. Schenzle, *Phys. Rev. A* **44** (1991) 1962.
 - [7] H. Nakazato, M. Namiki, S. Pascazio and H. Rauch, *Phys. Lett. A* **199** (1995) 27.
 - [8] W. H. Zurek, *Physics Today* **44**, No.10 (1991) 36; A. Venugopalan, D. Kumar and R. Ghosh, *Physica A* (to appear); A. Venugopalan, D. Kumar and R. Ghosh, *Current Science* **68** (1995) 62; A. Venugopalan, *Phys. Rev. A* **50** (1994) 2742.
 - [9] V. F. Weisskopf and E. Wigner, *Z. Phys.* **63** (1930) 54.

This figure "fig1-1.png" is available in "png" format from:

<http://arXiv.org/ps/quant-ph/9506029v3>